**Al-FARABI KAZAKH NATIONAL UNIVERSITY**

**Faculty of Mechanics and Mathematics**

**Department of Mathematical and Computer Modeling**

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|  | **Approve**  **at a faculty academic council meeting**  **The transactions No , 2015**  Dean of Faculty M.A. Bektemesov |

**SYLLABUS**

## Optimization Methods and Models (OM & M)

**The magistrates, Course 1, English,** **a** **first half-year, 6 credits**

**Professor: Kanat Shakenov,**

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**room: 319**

**Goals and objectives.** I believe that modern computers can play a similar role in mathematics. This course presents the innovative approach that numerical methods should be considered as a practical laboratory for undergraduate mathematics courses. I think this is innovative because it is not the state of affairs we currently encounter. On the one hand, first-, second- and third-year students in mathematics, science, and engineering learn introductory mathematical concepts without making appropriate use of computer technology. On the other hand, upper-level courses on numerical methods put their emphases on specific topics such as computational algorithms, error analysis, convergence and stability, and coding and debugging procedures, with only passing references to the mathematical background, which is generally assumed to be known and understood beforehand.

Differential equations describe the evolution of various quantities in many physical, chemical, biological, and economical problems. When the quantities depend on one variable, such as time, and do not depend on other variables, such as spatial coordinates, they may satisfy certain relations between the function and its first- and higher-order derivatives called ***ordinary differential equations****,* or simple***ODEs****.* Depending on the applied context of the problem, the only independent variable can have different meanings, for example, time, a space coordinate, a parameter. When the ODEs are used to model the evolution dynamics of a physical system starting with a given initial state, it makes sense to call the independent variable the *time variable,* as we do in this lecture. Two mathematical problems are associated with ordinary differential equations depending on whether the system of ODEs is supplemented by an ***initial*** value of the function at a single instance of the independent variable or its ***boundary*** values at distinct instances of the independent variable. This lectures covers existence and properties of solutions of the initial-value problems for ODEs, algorithms and errors of their numerical approximations, and an interplay between the numerical algorithms and their convergence, stability, and robustness.

***Boundary-value Problems for ODEs and PDEs (Partial Differential Equation).*** Whereas solutions of initial-value problems for ordinary differential equations (ODEs) exist whenever the vector field for the differential equation is sufficiently smooth, boundary-value problems may have no solutions if the boundary conditions are inconsistent. Nevertheless, many boundary-value problems arise in the context of physical and engineering problems where the existence of solutions can be complicated, it is legitimate to develop numerical approximations and graphical visualizations of the solutions before analyzing the properties of the given boundary-value problem, consistent with general strategy adopted in our numerical laboratory. Many specific results on solutions of boundary-value problems for ODEs and PDEs cannot be formulated as general theorems. Moreover, no general numerical recipe is available for numerical solutions of nonlinear differential equations. Experience and knowledge of various numerical routines are valuable assets in the design of new numerical algorithms for solutions of boundary-value problems. This lecture covers the simplest numerical approximations of solutions of boundary-value problems associated with ODEs and PDEs, as well as convergence, stability, and robustness of the numerical algorithms.

Trans-property: Algebra, Geometry, Mathematical analysis, ODE, PDE, Functional analysis, Integral equations, Computer science, Discrete mathematics, Algebra and Analysis Numerical Methods

Post-property: Mathematical Modeling, Computer Modeling, Numerical Mathematics, Numerical Fluid Mechanics, Numerical Methods of the Solution ODEs, Numerical Methods of the Solution PDEs, Hydrodynamics, Filtration Process.

**The structure of the course.**

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| --- | --- | --- | --- | --- |
| **Weeks** | **Name of subject (theme)** | **duration** | **Students self-instruction (SSI) by subject** | |
| Module 1. The Methods of Optimization of one Variables Function. | | | | |
| 1  -  2 | **Lecture 1-4.** Statement of problem.Classical Method. Bisection Method. Golden Section Method. Symmetrical Methods. Statement of problem of optimization methods. Optimization passive methods. Optimization step-by-step methods.  **Laboratory research 1-4.** Broken Method. Passive and Step-by-Step Selection Methods. Tangent Method. | 4  4 | | SSI-1-4  Convex function of one variables. |
| 3  -  4 | **Lecture 5-8.** Approximations of the normalized spaces. Outer and internal approximations. Discrepancy (disparity, residual), residual function. Approximation error. Discrete residual. Stability, stable operator. Convergence, discrete convergence.  **Laboratory research 5-8.** Discrepancy (disparity, residual), residual function. Approximation error. Discrete residual. Stability, stable operator. Convergence, discrete convergence. | 4  4 | | **SSI-5-8**  Approximation error. Discrete residual. Stability, stable operator. Convergence, discrete convergence. |
| Module 2. Numerical solution of LAES by Optimization Methods | | | | |
| 5  -  6 | **Lecture 9-12.** Systems of Linear Equations.Matrix Theory.Vector and Matrix Norms. Iterative Methods. General Schemes. The Sufficient Condition of convergence of the Iterative Process. Estimated Error. The Necessary and Sufficient Conditions of convergence of the Iterative Process.  **Laboratory research 9-12.** Iterative Methods. General Schemes. The Sufficient Condition of convergence of the Iterative Process. Estimated Error. The Necessary and Sufficient Conditions of convergence of the Iterative Process. Computer Aided Management (CAM) system programming. | 4  4 | | **SSI-9-12**  Vector and Matrix Norms computing. Iterative Methods. General Schemes. |
| 7  -  8 | **Lecture 13-16.** Numerical solution of LAES by Minimal Residual (Discrepancy, Disparity) Methods and convergence. Method of Steepest Descent and convergence.  **Laboratory research 13-16.** Numerical solution of LAES by Minimal Residual Methods. Method of Steepest Descent. | 4  4 | | **SSI-13-16**  Convergence of Minimal Residual and Steepest Descent methods. |
| 7 | **Midterm (Weeks 1 – 7 )** | 2 | |  |
| **Module 3. Numerical solution of ODE and PDE by Optimization Methods** | | | | |
| 9  -  10 | Lecture 17-20. Computational Methods Solution of the Boundary-Value Problems for ODEs. Boundary-Value Second-Order ODE Problem. Galerkin's method. Theorem 1. Complete orthonormal basis. Least-squares method. Shooting Methods for ODEs.  Laboratory research 17-20. Galerkin's method. Complete orthonormal basis. Least-squares method. Shooting Methods for ODEs. | 4  4 | | **SSI-17-20**  Optimization methods for boundary value second-order ODE Problems. |
| 11  -  12 | Lecture 21-24.Elliptic PDE Problem. Dirichlet boundary-values problem for Poisson equation. Direct method. Variational Methods for PDEs. Ritz method. Theorem 1. Characterization of the Ritz methods. Definition 1. Theorem 2.  **Laboratory research 21-24.** Ritz method. Theorem 1. Characterization of the Ritz methods. Definition 1. Theorem 2. | 4  4 | | **SSI-21-24**  Gradient Method. Gradient Projection Method. Subgradient Projection Method. Condition Gradient Method. Feasible Direction Method. Conjugate Direction Method. Newton Method. Cubic Rate of Convergence Method. Coordinate-wise Descent Method. |
| 13  -  14 | Lecture 25-28. Solution of the Diffusion difference equation by Ritz methods. Sweep Method Solution of Diffusion difference equation. Stability. Gradient Method. Gradient Projection Method. Subgradient Projection Method. Condition Gradient Method. Feasible Direction Method. Conjugate Direction Method. Newton Method. Cubic Rate of Convergence Method. Coordinate-wise Descent Method. **Laboratory research 25-28.** Barrier Functions Method. Loaded Functions Method. | 4  4 | | **SSI-25-28**  Barrier Functions Method. Loaded Functions Method. Random Search Method. |
| 15 | **Lecture 29-30.** Subgradient Projection Method. Condition Gradient Method. Feasible Direction Method. Conjugate Direction Method. Newton Method. Cubic Rate of Convergence Method. Coordinate-wise Descent Method. Laboratory research 29-30. Random Search Method. | 2  2 | | **SSI-29-30**  Feasible Direction Method. Conjugate Direction Method. Newton Method. Cubic Rate of Convergence Method. Coordinate-wise Descent Method. |
| 16 | **Exam** | 2 | |  |

**References**

**Basic:**

1. James M. Ortega and Werner C. Rheinboldt. ITERATIVE SOLUTION OF NONLINEAR EQUATIONS IN SEVERAL VARIABLES. Academic Press. New York and London 1970.
2. F.P. Vasiliev. Numerical Methods of the Solution of Extremes Problems. Nauka. Moscow, 1980. (In Russian).
3. B.N. Pshenichnyi, U.M. Danilin. Numerical Methods in Extremes Problems. Nauka. Moscow, 1975. (In Russian).
4. K.K. Shakenov. Numerical Mathematics Methods. Lecture Courses. Print S. Almaty, 2009. (In Kazakh).
5. Matheus Grasselli, Dmitry Pelinovsky. Numerical Mathematics. Narosa Publishing House. India. 2009.
6. George E. Forsythe, Wolfgang R. Wasow. FINITE-DIFFERENCE METHODS FOR PARTIAL DIFFERENTIAL EQUATIONS. JOHN WILEY & SONS, INC. NEW YORK – LONDON, 1959.
7. Robert D. Richtmyer, K.W. Morton. DIFFERENCE METHODS FOR INITIAL – VALUE PROBLEMS. Second Edition. INTERSCIENCE PUBLISHERS a division of John Wiley & Sons. NEW YORK LONDON SYDNEY, 1967.
8. JEAN-PIERRE AUBIN. APPROXIMATION OF ELLIPTIC BOUNDARY-VALUE PROBLEMS. WILEY-INTERCSIENCE a Division of John Wiley & Sons, Inc. New York London Sydney Toronto, 1972.
9. C. A. J. Fletcher. COMPUTATIONAL GALERKIN METHODS. Springer-Verlag. New York Berlin Heidelberg Tokyo, 1984.

**Additional:**

1. V.M. Alexeev, V.M. Tihomirov, S.V. Fomin. Optimal Control. Nauka. Moscow, 1979. (In Russian).
2. V.I. Zubov. Lecture of the Control Theory. Nauka. Moscow, 1975. (In Russian).
3. N.N. Moiseev, U.P. Ivanilov, E.M. Stoliarova. Optimization Methods. Nauka. Moscow, 1975. (In

Russian).

1. R. W. HAMMING. NUMERICAL METHODS FOR SCIENTISTS AND ENGINEERS. MC

GRAW-HILL BOOK COMPANY. INC. NEW YORK, SAN FRANCISCO, TORONTO, LONDON,

1962.

1. WILLIAM EDMUND MILNE. NUMERICAL CALCULUS. Approximations, Interpolation, Finite

Differences, Numerical Integration and Curve Fitting. PRINCETON UNIVERSITY PRESS.

PRINCETON, NEW JERSEY, 1949.

1. KAISER S. KUNZ. NUMERICAL ANALYSIS. McGraw-Hill Book Company, Inc. NEW YORK

TORONTO LONDON, 1957.

1. Roger TEMAM. NAVIER – STOKES EQUATIONS. Theory and numerical analysis. Revised

**Control Test: twice.**

**SSI***:* **a few times.**

**Criterion of grade of the knowledge, marks in percent**

|  |  |
| --- | --- |
| *Lecture* | *30 +*  *30* |
| *SSI – theory* |
| *SSI – practice(seminar)* | *30* |
| *Total written examination* | *90* |

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Weeks** | **Lecture** | | **Seminar** | | **SSI** | | **TC** | **TOTAL** |
|  | **No.** | **Mark** | **No.** | **Mark** | **No.** | **Mark** |  | **Mark** |
| 1 | 1 | 0,5 | 1 | 2 | 1 | 0,5 |  | 3 |
| 2 | 2 | 0,5 | 2 | 2 | 2 | 0,5 |  | 3 |
| 3 | 3 | 0,5 | 3 | 2 | 3 | 0,5 |  | 4 |
| 4 | 4 | 0,5 | 4 | 4 | 4 | 0,5 |  | 5 |
| 5 | 5 | 0,5 | 5 | 5 | 5 | 0,5 |  | 6 |
| 6 | 6 | 0,5 | 6 | 3 | 6 | 0,5 |  | 4 |
| 7 | 7 | **-** | 7 | 3 | 7 | **-** | 3 | 6 |
| **Total: Weeks 1-7** |  | **3** |  | **21** |  | **3** | **3** | **30** |
| 8 | 8 | 0,5 | 8 | 2 | 8 | 0,5 |  | 3 |
| 9 | 9 | 0,5 | 9 | 4 | 9 | 0,5 |  | 5 |
| 10 | 10 | 0,5 | 10 | 3 | 10 | 0,5 |  | 4 |
| 11 | 11 | 0,5 | 11 | 3 | 11 | 0,5 |  | 4 |
| 12 | 12 | 0,5 | 12 | 2 | 12 | 0,5 |  | 3 |
| 13 | 13 | 0,5 | 12 | 1 | 13 | 0,5 |  | 2 |
| 14 | 14 | - | 12 | 3 | 14 | - |  | 3 |
| 15 | 15 | - | 12 | 3 | 15 | - | 3 | 6 |
| **Total: Weeks 8-15** |  | **3** |  | **21** |  | **3** | **3** | **30** |
| **Total: Weeks 1-15** |  | **6** |  | **42** |  | **6** | **6** | **60** |

**The scale of mark of knowledge:**

|  |  |  |  |
| --- | --- | --- | --- |
| **Letter symbol of mark** | **Digital of mark (GPA)** | **Mark (percent)** | **Mark on tradition system** |
| A | 4 | 95-100 | “excellent” |
| A- | 3,67 | 90-94 |
| B+ | 3,33 | 85-89 | “good” |
| B | 3 | 80-84 |
| B- | 2,67 | 75-79 |
| C+ | 2,33 | 70-74 | “satisfactory” |
| C | 2 | 65-69 |
| C- | 1,67 | 60-64 |
| D+ | 1,33 | 55-59 |
| D | 1 | 50-54 |
| F | - | 0-49 | “unsatisfactory” |
| I | - | - | “Incomplete discipline” |
| W | - | - | “Renunciation of discipline” |
| AW | - | - | “Deduction off discipline” |
| AU | - | - | “To take a discipline” |
| P/NP (Pass / No Pass) | - | 65-100/0-64 | “Pass / No Pass” |

Sitting of the chair consideration

Protocol No. , , 2015

**Acting as chief of chair M & C M,**

PhD, docent \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Dauren Zhakebaev

**Lecturer**

Doctor of Science, Professor \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Кanat Shakenov